

We are still learning to integrate affect (and mathematics) into our research

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Mathematics enables us to investigate, explain, and make sense of the world in which we live (Ministry of Education, 2007). Many people, however, are unable to do this fully because of their affective views and responses to mathematics. In this paper, a review of affective research in mathematics education is presented to provide a context for exploring research that includes affective aspects at MERGA conferences over the last 25 years, and to position my own research. Researchers of affect are challenged to maintain a focus on mathematics, and researchers working in the broader field of mathematics education are challenged to incorporate affective aspects into their research.

Introduction

We conduct research in mathematics education to improve students' learning of mathematics. This goal is embedded in the policies of the Mathematics Education Research Group of Australasia (MERGA) and is a worthwhile aim on which to spend a lifetime's work. Indeed, in the 40-year life of MERGA our community has learnt a lot about research, students, learning and mathematics. We are still learning, and the process of challenging ourselves helps to ensure this continues. In this paper, I seek to challenge myself as an affective researcher, but also to challenge others, whether they consider themselves to be an affective researcher or not.

During my membership of MERGA several challenges from MERGA members have particularly resonated with me. Tom Lowrie (2015) discussed the implications of the trend towards non-mathematicians in MERGA, which led me to query my own background as a researcher. This is linked to Robyn Jorgensen's (2014) caution that the focus in mathematics education research seems to have shifted from the core learning of mathematics to the social conditions within which learning occurs. In turn, this resonated with me because of its connections with what Bill Barton (2003) said when he underlined the importance of connecting affective research with students' learning. "We should be careful about doing research that is easy, rather than research that contributes to our understanding of mathematics learning" (p. 85). Conversely, Andy Begg (2000) has regularly reminded me that the role of a teacher is to teach students, rather than mathematics, and I need to extrapolate this wisdom about teaching to my role of mathematics education researcher.

In this paper, I first outline affective research in mathematics education. This is a broad-brush approach only; recent exhaustive reviews of affective research can be found elsewhere (see Attard, Ingram, Forgasz, Leder, & Grootenboer, 2016; Goldin et al., 2016; Grootenboer & Marshman, 2016). I will map affective research presented at MERGA conferences over the last 25 years, and use these understandings to provide a context and critique for my own research and my own developing identity as both an affective researcher and a mathematics education researcher.

Researching mathematical affect

Affect is an umbrella term used to describe a range of aspects of the human mind that go beyond cognition, such as beliefs or the experience of feelings and emotions (Hannula, 2012; McLeod, 1992). Affect is well established as an integral part of students' processes of learning mathematics (Op 't Eynde, De Corte, & Verschaffel,

2006). When an individual engages in mathematics, there is more going on than the application of logic and reasoning; engagement is mediated by the individual's affective views and responses.

Researchers working in the field of mathematics education have a variety of reasons for the focus on or inclusion of affective factors in their research. There is compelling evidence that a person's confidence and proficiency in applying their mathematical knowledge impacts not only their mathematical learning, but also their career opportunities and participation in our technologically-rich society (Anthony & Walshaw, 2009). Declining engagement within school (Sullivan, McDonough, & Harrison, 2004), and declining numbers of students participating in mathematics in the senior school when it is no longer compulsory, and as a major at university, are factors that often preface research (e.g., Brown, Brown, & Bibby, 2008). Assisting primary preservice teachers (e.g., Ingram & Linsell, 2014), or those undertaking mathematics minors at university (e.g., Nardi, 2016), to have robust mathematical content knowledge are also cited as catalysts for affective research in mathematics education. Perhaps most importantly, many students do not enjoy mathematics (Evans, 2000) and some researchers assume that a person's positive affect is a worthwhile outcome in itself for research (e.g., Falsetti & Rogríguez, 2005).

Grootenboer and Marshman (2016) describe the researching of affect in mathematics education as a "contested space" (p. 13). Certainly it is a busy one. Affective researchers in mathematics education come from a variety of backgrounds. These include psychology, sociology, neurophysiology, and education. There is resulting variety, and at times, muddiness in researchers' focus and theoretical perspectives. Affective researchers in mathematics education have variously explored participants' beliefs, attitudes, values, identity, self-efficacy, moods, norms, goals, confidence, emotions, anxiety, and motivation, and they have explored these factors in relation to each other as well as gender, ethnicity, socio-economic status, engagement, achievement, and participation. The affective views and responses of a range of participants have been studied, although studies of school students and preservice teachers dominate (Attard et al., 2016).

Early affective research was initiated by social psychologists in the 1970s (Hannula, 2014) and centred on studies of mathematical anxiety and attitude, and these types of studies continue (e.g., Jennison & Beswick, 2010; Wilson, 2009). *Mathematics anxiety* is often described (e.g., Ashcraft & Moore, 2009) as "a feeling of tension and anxiety that interferes with the manipulation of numbers and the solving of mathematical problems in a wide variety of ordinary life and academic situations" (Richardson & Suinn, 1972, p. 551). *Mathematics attitude* is usually defined as a general liking or disliking of mathematics and a predisposition to respond in a favourable or unfavourable way (Hart, 1989). Early researchers generally had an atheoretical approach with varying and overlapping definitions for these constructs, making it somewhat difficult to interpret and compare results. Affective variables were studied to prove causal correlations between the constructs and other measures such as achievement. Usually measured by self-report questionnaires, these became more sophisticated over time, moving from small scale studies designed to measure a single affective dimension (e.g., Aiken & Dreger, 1961) to large-scale multivariate investigations (e.g., Chiu & Henry, 1990). There is robust critique in the affective literature of quantitatively analysed, large-scale studies because of their perceived inability to fully describe student experience (see Grootenboer & Marshman, 2016). Open-ended surveys and interviews as well as mixed-methods have, in latter years, been widely employed (e.g., Boyd, Foster, Smith, & Boyd, 2014). Mathematics

anxiety and attitude has variously been related to general and test anxiety, gender and achievement (Hembree, 1990; Ma & Xu, 2004). However, whether the direction of influence between affect and achievement is negative or positive is often unclear or unexplored, and there has been a shift from a causal-relationship paradigm to an interpretative one (Goldin et al., 2016).

McLeod's (1992) conception of an affective domain dominated an era of affective research related to problem solving. He defined *affect* as a "wide range of beliefs, feelings, and moods that are generally regarded as going beyond the domain of cognition" (p. 576). There were three elements in this conception: beliefs, attitudes, and emotion. This conceptualisation was significant because it captured some of the complexity of a person's affect and allowed for relationships between elements to be explored; important in order to understand students' mathematical learning (Grootenboer & Marshman, 2016). Beliefs, attitudes and emotions were conceptualised as varying in stability, intensity and cognitive involvement; a person's emotions change rapidly but a person's beliefs are cool, stable constructs that develop over a relatively long period of time. *Beliefs* are regarded as an individual's subjective conceptions, or anything that they regard as true (Beswick, 2007). Beliefs, although somewhat cognitive in nature (Hannula, 2012) can, arguably, be distinguished from *knowledge*, which is shared, external and accepted by the mathematics community (Furinghetti & Pehkonen, 2002). The beliefs students have about mathematics, largely shaped by students' experiences in the mathematics classroom, have implications for student behaviour and "extraordinarily powerful" consequences for affect and learning (Schoenfeld, 1992, p. 359). *Emotions* were generally described as hot reactions to mathematics, occurring when there is a discrepancy between the individual's expectations and the demands of an on-going activity (Mandler, 1989). Experiencing emotion is thought to lead to a reduction in the conscious capacity available because the process of emotional construction itself requires conscious capacity. Emotions can therefore bias attention and memory, and may activate actions as students reflect on and try to control them (Mandler, 1989).

Debate continues about the inclusion of the elements in McLeod's conceptualisation. It is not clear where, for example, concepts such as motivation and identity fit. Similar to the earlier work into anxiety and attitude, the research lacks sufficient theorising, the terms are often ill defined, or the definitions are only implicitly implied through research instruments. Furthermore this conceptualisation emerged from relatively slow problem solving tasks; there is little "fine-grained" (Ashcraft, 2001, p. 224) examination of mental representations and processes in this conceptualisation (see Hannula, 2012 for further critique).

Hannula (2012) more recently captured the complexity of the affective domain through proposing a meta-theory of mathematics-related affect (see figure one). He included three distinct dimensions of affect:

1. Cognitive (beliefs), motivational and emotional aspects. Beliefs deal with information about the self and the environment, and motivation directs behaviour. Success and failure are reflected by emotions, which provide feedback about cognitive and motivational processes;
2. Movement between state and trait, where state refers to rapidly changing affective states when faced with a mathematical situation and traits are relatively stable affective tendencies;
3. Physiological (embodied), psychological (individual) and social theories of affect (Goldin et al., 2016; Hannula, 2012).

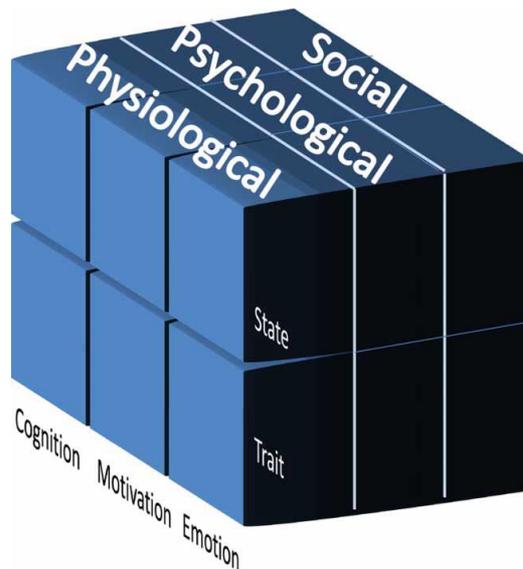


Figure 1. The three dimensions of mathematics-related affect (Hannula, 2007).

This useful meta-theory enables connections to be made across different theories of mathematical-related affect and across different eras of research in the affective domain. It also highlights aspects that have not received sufficient attention; Hannula (2012) identifies that traits have been focused on over states, and psychological studies have been focused on over others. Physiological theories of affect are rarely considered in mathematics education, although there is extensive neuropsychological research on mathematics anxiety (Goldin et al., 2016).

Affective research presented at MERGA Conferences 1992-2016.

Given affect has been established as an integral part of mathematical learning for 25 years, I sought to find out how established it is as a research domain within the MERGA community. I chose to map research presented at MERGA conferences because participation in the conference is the enactment of our community. I did not review the findings; there are already extensive reviews on affective research in mathematics education within our region over this time period (e.g., Attard et al., 2016). Rather, my scope was to find out the extent to which our community has taken affect into account, what affective research has been done and by whom.

For a researcher/paper to be included in this mapping, there needed either to be emphasis in the title or abstract on aspects of affect, or affective aspects had to be noted in the findings or implied by the reference list. For each affective paper since 1992, I noted the researcher, the affective construct/s used, the research participants, and I noted any affective literature informing the research. I have not included short communications or round tables because abstracts often did not tell the whole story and references were needed to understand the depth of the affective focus. This means some affective research has not been included, such as the tantalising round table on emotional engagement by Higgins and Bobis in 2015.

Findings. In the last 25 years, 10.4% (202) of the full research papers included in the conference proceedings and presented at MERGA conferences have included an affective aspect. In other words, studies in mathematics education that relate to affect are relatively small in proportion to more cognitively-oriented studies. 42% of the affective research reports focused on beliefs (the next most prevalent aspect was attitudes at around 10%). There was a full range of participants involved over the 25 years with a notable exception of early childhood students and teachers. Research

involving primary preservice teachers dominated. These findings are similar to wider reviews of affective research in mathematics education (e.g., Goldin et al, 2016).

Of note is the quality of affective research. It is pleasing that 79% of the affective research reports were grounded by affective literature. Those research reports that did not refer to this body of literature usually related to intervention studies (e.g., what is the effect of X on attitudes) or the use of research tools to capture students' affect (e.g., capturing students' perceptions/beliefs/views of mathematics using X). These research papers often contained affective terms that were not adequately defined for research to be compared. Affective research needs to have constructs that are carefully defined and located within the wider body of affective literature.

The last 20 years of affective research in mathematics reflect, to varying extents, the growing interest and focus on the social and cultural context of the classroom (Sfard & Prusak, 2005), and focus on theories that see meaning, thinking, and reasoning as social products (Lerman, 2000). Again, this social turn in mathematics education is reflected by the research presented at MERGA. Jorgensen (2014) cautioned that these social perspectives have meant that focus is shifting away from the core learning of mathematics, and there is evidence the affective literature at MERGA has contributed to this. Too often, mathematics is dealt with very generally and there is a sense that the word 'mathematics' could be removed and replaced with another subject such as 'physics' or 'economics' without much loss of understanding.

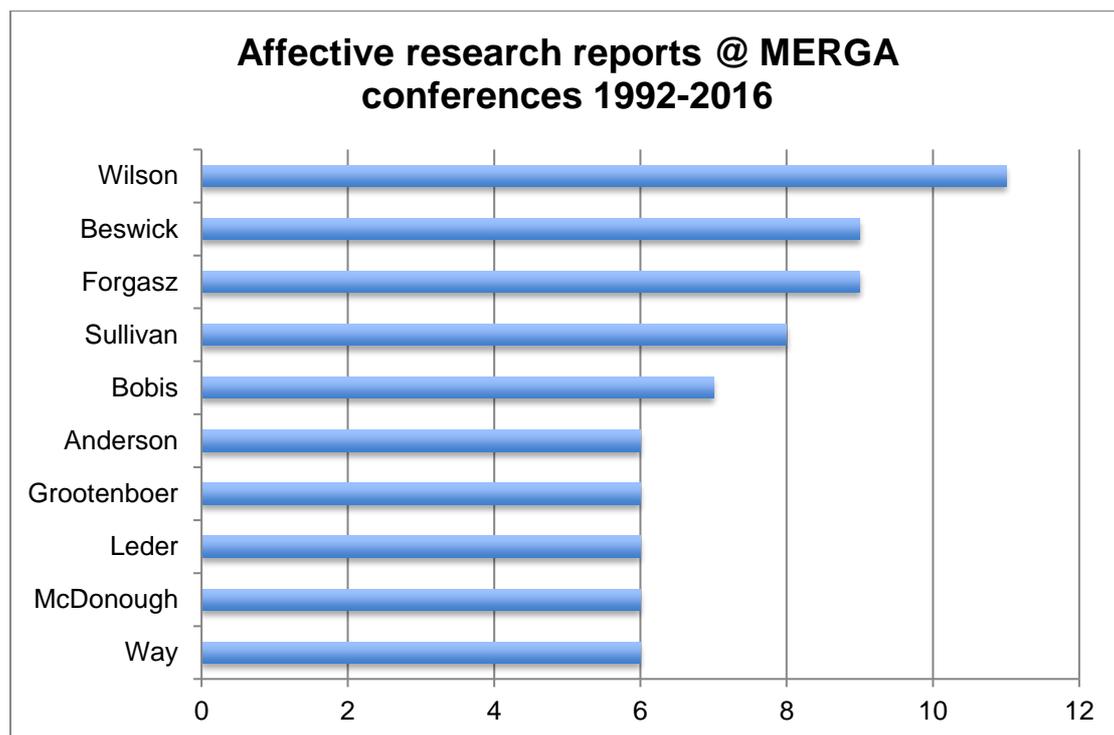


Figure One: Affective research reports at MERGA conferences 1992-2016

There have been 184 researchers present papers that include affective aspects since 1992. The five most active of these (see Figure One) study quite separate aspects of affect, however their commonalities are notable in terms of the literature informing their research. They are mostly informed by the research of their MERGA colleagues (Beswick, Forgasz, Leder, Sullivan and Grootenboer), and the research of McLeod, Hannula, Ernest, Pajares, Boaler, Ma, and Thompson.

Another commonality of the most active affective researchers is that they all began their career as teachers of mathematics; Bobis as a primary teacher and the

others in the secondary sector. As Lowrie (2015) highlighted, the discipline of mathematics education has now stronger foundations within education than it does within mathematics. Looking at the list of affective MERGA researchers, this seems true in this sub-field also. These affective researchers may or may not consider themselves to be mathematicians, psychologists or neurophysiologists. However, they are experienced researchers and classroom teachers in the field of mathematics education and as such they operate effectively at the nexus of all of these disciplines. They are well positioned to undertake research in this field.

My own research

My own background is also in mathematics teaching. I am an experienced secondary mathematics teacher, with a postgraduate mathematics degree and a doctorate in mathematics education. I am currently a researcher and lecturer in secondary and primary mathematics education. Lowrie (2015) lamented the lack of mathematicians in our research community, and although I do not teach mathematics at a tertiary level, I do consider myself to be a mathematician, but I have never formally studied psychology, physiology or sociology. As with the more prolific affective researchers, I have experience that psychologists, physiologists and sociologists may not have. I fulfill my role armed with my knowledge of mathematics, the New Zealand school mathematics curriculum, effective pedagogy of mathematics (Anthony & Walshaw, 2009), my experience in the classroom, my knowledge of New Zealand's unique context, and my research-based understanding that each student has a unique relationship with mathematics.

I am also informed by the research of others. My main influences are a range of researchers working within and beyond the affective domain. These researchers are my touchstones and I situate my own work within the context of this wider research (see Figure 2).

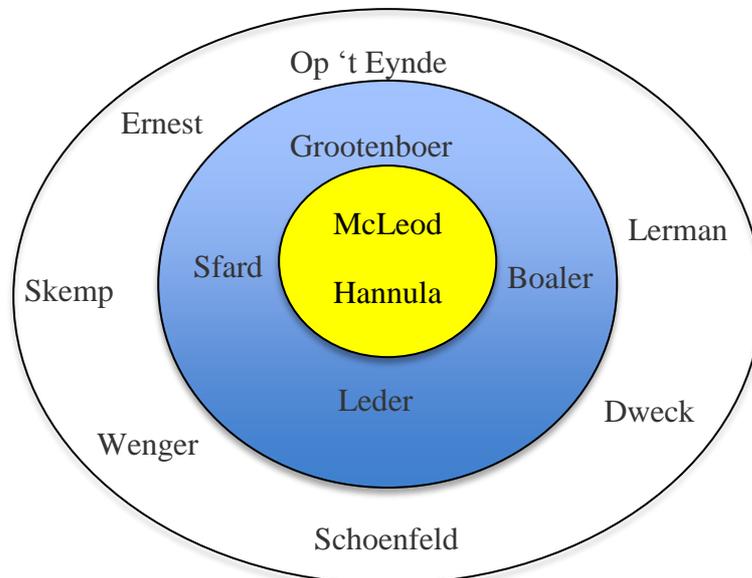


Figure 2. My research influences

This background of affective literature provides a context for my own research. I either focus on the affective domain, or incorporate an affective aspect into my work. My main research thus far has been the study of mathematical journeys of a group of 31 adolescents. I collated data on these students from Years 7-12, but studied them intensely over two years, when they were aged 13-16 (Years 10 and 11 in New Zealand). During this period, I observed them in their mathematics and English classes, using video and audiotapes to capture their voices and facial expressions. I

interviewed the students, their teachers, and surveyed their parents. I further captured students' views of mathematics through metaphor, drawings of mathematicians and personal journey graphs. This large collection of data continues to occupy me, and has resulted in a number of publications (Ingram, 2007, 2008, 2009, 2011, 2013).

In other affective research, I have completed several iterations of a project in which the use of Show and Tell tablet technology applications have been investigated in primary and secondary mathematics classroom. Particular attention was paid to the quality of students' engagement during problem solving (e.g., Ingram, Williamson-Leadley, & Pratt, 2015). In an iteration of the Encouraging Persistence, Maintaining Challenge Project, I was part of a team that included an exploration into how teachers encourage students to persist in challenging angles tasks (Ingram, Holmes, Linsell, Livy, & Sullivan, 2016). I am currently in the midst of a longitudinal study of preservice primary teachers called "Growing Mathematics Teachers" that aims to understand how preservice teachers' identities as teachers of mathematics develop.

It has been a privilege to engage in this research and to discover for myself the connections between affect and mathematical learning. Learning mathematics is, to me, change in both the cognitive and affective elements of an individual's relationship with mathematics. A summary of my understandings is described below. These understandings locate me among the affective researchers as having a broad and dynamic view of affect.

- 1. Individuals have unique relationships with mathematics.** The school students in my longitudinal study described relationships with mathematics with five interacting elements: views related to mathematics, macro-feelings¹ about the subject of mathematics, mathematical knowledge, identities related to expectations of being able to do the mathematics, and habits of engagement, including engagement skills. These contributed to the context within which they engaged in mathematics. This construct of a relationship is similar to constructs in affective literature called mathematical self-systems (Malmivuori, 2006), identities or dispositions (Op 't Eynde et al., 2006), which variously include: mathematical content knowledge (Malmivuori, 2006); beliefs about mathematics (Op 't Eynde et al., 2006); related goals (Csikszentmihalyi, 1988) and needs of autonomy, competency, and social belonging (Hannula, 2006); global affects or attitudes; meta-knowledge, which involves knowledge about meta-cognitive functioning and affect (Malmivuori, 2006); and, habitual, re-current affective pathways and behaviours in mathematics (Goldin, 2002).
- 2. When a student is exposed to a mathematical situation, each student interprets the situation according to his or her unique relationship with the subject.** The student may assess what knowledge may be needed for the task, if they have that knowledge and how accessible it is (Malmivuori, 2006). Students have varying interpretations of how well they will be able to complete the task, whether it adheres to their view of what the nature of mathematics is or the type of task they might expect to get in that class. A student interprets the mathematical situation according to their *context of the moment*; the current context in which they find themselves. They also interpret the situation according to their macro-feelings about the subject of mathematics overall and they may experience a level of motivation because doing it will help them to fulfill their goals (Hannula, 2012).

¹ These students had just been introduced to macro and micro-economics in social studies.

As a result of these complex interpretations, the individual experiences a wide range of affective responses. The students in my research called these micro-feelings, but in the literature they are referred to as local affects (Goldin, 2002) or state affects (Hannula, 2012). These affective responses could be hot emotions with accompanying physiological arousal such as anxiety or joy; or less hot responses such as boredom or interest. The student's interpretation of the response may then disrupt or distract the learning process and affect the level of capability while engaging in the task (Mandler, 1989).

3. **The student engages in the mathematical task.** The context of the moment, and aspects of the student's relationship with mathematics, including their engagement skills, mediates the level and quality of engagement. The individual continues to experience micro-feelings during their engagement, and these affective responses are interpreted and perhaps acted on by the student. DeBellis and Goldin (2006) usefully described a positive affective pathway as one where the student begins by experiencing curiosity and puzzlement if the problem is unfamiliar and difficult. They are motivated to better understand the problem. As the problem solving continues, the person goes through a stage of bewilderment and frustration, which carries the meaning that the strategies employed so far have led to insufficient progress. One or more changes of strategy eventually yield pleasure and satisfaction. In a negative pathway, frustration does not lead to a change of strategy and ends in the student experiencing anxiety and despair, which evokes avoidance strategies and defense mechanisms.
4. **Students have unique performances and learning experiences.** These experiences are further interpreted in relation to his or her relationship with mathematics and these interpretations reinforce or, if sufficiently powerful or repeated often enough, alter their dynamic relationship. Students who have completed a task successfully may have expected to do so. Others may have given up quickly and do not attempt to understand it further. There will be little change in aspects of their relationship with mathematics as a result. Some students may have completed a task successfully, after several attempts, gaining new knowledge and gaining confidence in that particular type of problem. Others may have faced particular trouble with the task, when normally they find mathematics easy. Their experience may have been powerful because their difficulty was in front of the class. For these students, elements of their relationship with mathematics may alter. New, important or personally significant mathematics learning experiences further build up or alter students' relationships with mathematics. These relationships with mathematics are therefore constantly changing and re-negotiated during every learning experience in the classroom.

An individual's doing of mathematics is rather like doing a long jump. Before approaching the pit, athletes have a view of long jump, which depends on their experience at previous events, whether it is compulsory to attend, and what the weather is like. When it is their turn for the long-jump, the athlete pauses at the mark; anticipating the jump, perhaps dreading it, perhaps thinking through the number of steps or their technique. They are aware of who is watching, where they are expected to land and where they probably will land. This whole plethora of complex factors remains with them as they jump, land and as they decide if their jump was successful. So it is with doing mathematics.

My research has had, thus far, a modest effect on the practice of teaching and learning mathematics. Through local and national mathematical associations, I

encourage teachers to get to know their students, understand their relationships with mathematics and monitor these relationships over time. I ask them to make affective aspects of learning mathematics explicit in their teaching, and to build students' engagement skills by embracing and normalising the confusion of doing mathematics and to reflect on their doing.

As Lowrie said, "mathematics colleagues are concerned with the decrease in the number of students wanting to undertake degrees with a mathematics specialization" (2015, p. 18). This is true for our university and, as an associate member of the mathematics department, I work to understand the decline of participation in mathematics as a major and to bridge the gap between school and university mathematics in terms of breadth and level of content knowledge of first year undergraduates.

Conclusion

It is timely to review the prevalence and influence of affective research at this point in MERGA's history. Just as learning mathematics is a social practice, socially and culturally constituted, so is learning in research. I may be isolated from the other researchers in this field by space and time, but by connecting with their research, I am participating in social practice.

When our social practice of learning involves questioning and querying what [has already been done], then individuals will find new ways to do things ... It is the social practice of learning through querying and questioning that enables the individual to produce new knowledge (Meaney, 2009).

Similarly to the word 'significant' in research, affective terms, such as belief and attitude need to be used with caution. When a researcher uses clear definitions of affect and their research is grounded in wider affective literature of the community and the field, this enables research to be compared to other research, utilised and built upon. Building on previous research is a way to ensure that that each generation of mathematics educators does not end up wrestling with many of the same problems the preceding generations thought they had "solved" (Kilpatrick, 1984, p. 45; highlighted previously by Leder, 1994)

I challenge affective researchers to remain, become or return to being mathematics education researchers, and I challenge others in the mathematics education field to integrate affect into their research. Mathematics is not a "purely intellectual endeavour, where emotion has no place" (Hannula, Evans, Philippou, & Zan, 2004, p. 109). I fail to see how a researcher can understand why students struggle with algebra in a fine-grained way without factoring in affect. However, the idea of affective research in our field where mathematics has no place is equally fraught. Some affective studies at MERGA and, at times, my own research, would not hold up well to the question: 'Could this finding apply to any other subject or is this unique to mathematics?' Research in mathematics education needs to include cognitive *and* affective aspects rather than cognitive *or* affective aspects to understand students' learning of mathematics. Researchers of affect are challenged to maintain a focus on mathematics, and researchers working in the broader field of mathematics education are challenged to incorporate affective aspects into their research.

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